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Directional photon transfer between two wires

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The directional transfer of a single photon from one wire to another, leaving all other neighbor states unaffected, is of great importance. We present a simple coupling structure that makes such transfer possible, for any given photon wavelength and linewidth. We give closed-form expressions for the parameters necessary to build such a structure. An illustration of our analytic study is given for the directional transmission of a telecommunication signal between two lines.

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The directional transfer of photons [1–3] between two wires has been intensively investigated in the recent years. Applications of such transfer processes are, in particular, important for wavelength multiplexing in communication devices.

In order to allow a directional transfer of one photon with a given energy between two monomode wires, one has to build an appropriate coupling structure to permit all incoming particles of different energies to continue to travel without perturbation along the input wire and to transfer successfully one to the output wire in a given direction. The criteria for such transfers were given on the basis of symmetry arguments and were illustrated by applications to structures designed in photonic materials by rod removal and perturbation [2]. We proposed other structures [3] made of resonating finite wires. All the above structures require very precise definition of their parameters.

In this paper we propose a simpler structure and give closed form expressions for all the parameters that are necessary for the device fabrication. Another advantage of the present structure is that it requires less tuning accuracy than the earlier ones.

Let us consider the system presented in Fig. 1. The two continua are the two infinite lines passing by, respectively, points (1,2) and (3,4). The distance d_0 between points 1 and 2 is the same as between points 3 and 4. Four identical monomode structures are branched between points (1,5), (5,4), (2,6), and (6,3). These structures have one finite wire of length d_2 branched in the middle of the lines of length $2d_1$ situated between the above-given points. The wave function associated with the particles traveling in this structure is supposed to vanish at the free end of the dangling wires. Such structures enable transmission gaps to be opened in the desired frequency range in the same manner as in similar stub structures [3]. Between points 5 and 6 is fixed a waveguide of length $3d_1$ with two finite wires of length d_2 in its middle, which acts as a resonant cavity with a localized mode in the above-mentioned gap.

The reflection and transmission coefficients as a function of wavelength λ associated with the particles traveling in the monomode wires were found to be given by the following expressions:

$$R=|z_1+z_2+z_3+z_4+1|^2, \tag{1a}$$

$$T_{12}=|z_1+z_2-z_3-z_4|^2, \tag{1b}$$

$$T_{13}=|z_1-z_2+z_3-z_4|^2, \tag{1c}$$

$$T_{14}=|z_1-z_2-z_3+z_4|^2, \tag{1d}$$

where

$$z_n=\frac{i}{2}(y_n-i)^{-1}, \quad n=1,2,3,4, \tag{2}$$

$$y_1=y_2-\frac{2B_1^4}{(2A_1+A_2)^2\left(3A_1-\frac{2B_1^2}{2A_1+A_2}-\frac{B_1^2}{2A_1+A_2+B_1}\right)}, \tag{3a}$$

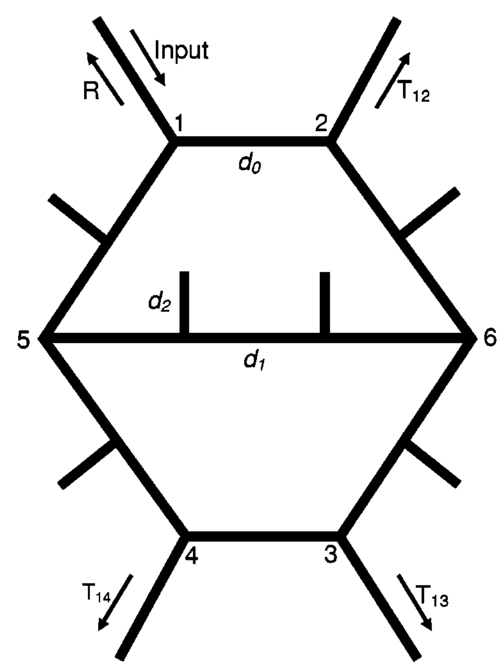

FIG. 1. The multiplexer design.

$$y_2=\tan\left(\frac{\pi d_0}{\lambda}\right)+A_1-\frac{B_1^2}{2A_1+A_2}, \tag{3b}$$

$$y_3=-\left[\tan\left(\frac{\pi d_0}{\lambda}\right)\right]^{-1}+A_1-\frac{B_1^2}{2A_1+A_2}, \tag{3c}$$

$$y_4=y_3-\frac{2B_1^4}{(2A_1+A_2)^2\left(3A_1-\frac{2B_1^2}{2A_1+A_2}-\frac{B_1^2}{2A_1+A_2-B_1}\right)}, \tag{3d}$$

$$A_i=-\left[\tan\left(\frac{2\pi d_i}{\lambda}\right)\right]^{-1}, \quad i=1 \quad \text{and} \quad 2, \tag{4a}$$

and

$$B_1=\left[\sin\left(\frac{2\pi d_1}{\lambda}\right)\right]^{-1}. \tag{4b}$$

We then find that, in order to obtain a complete transfer of a particle with a propagating vector $k_0=2\pi/\lambda_0$ from site 1 to site 3 (namely, $T_{13}=1$ and $R=T_{12}=T_{14}=0$), the structure given in Fig. 1 must be such that, for $\lambda=\lambda_0$,

$$y_1=y_3=-\frac{1}{y_2}=-\frac{1}{y_4}. \tag{5}$$

The above expressions provide the following solution:

$$d_0=(1+4n_0)\frac{\lambda_0}{4}, \tag{6a}$$

$$d_1=(1+4n_1\mp\delta)\frac{\lambda_0}{4}, \tag{6b}$$

and

$$d_2=(2n_2\pm\delta)\frac{\lambda_0}{4}, \quad n_i=0,1,2,3,\ldots, \tag{6c}$$

where

$$\delta=\left[\frac{3(1+4n_1+2n_2)}{\pi Q}\right]^{1/2}, \quad n_i=0,1,2,3,\ldots \tag{7}$$

is supposed to be small compared to 1 and Q is the quality factor associated with the linewidth of the transferred signal defined by

$$Q=\frac{\lambda_0}{2(\lambda'-\lambda_0)}, \tag{8}$$

where λ' is the wavelength at which $T_{13}(\lambda')=0.5$.

In the same manner, we define a quality factor Q' for the envelope of the direct transmission T_{12} (not to be confused with the dip in T_{13} due to the transferred particle):

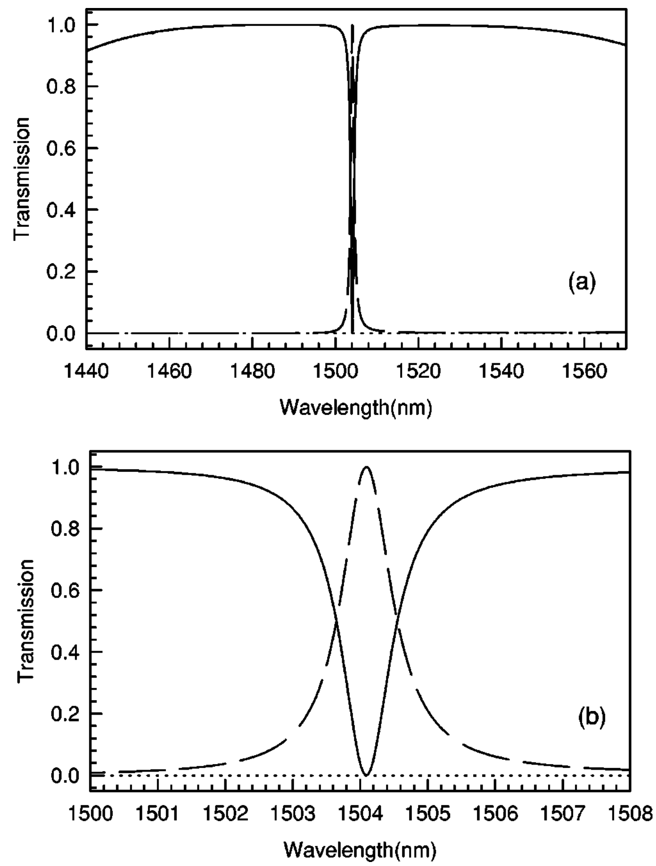

FIG. 2. Variation of the intensity of the transmitted signal T_{12} from site 1 to site 2 (solid line), and of the forward signal T_{13} (long dashed line), in the structure shown in Fig. 1 versus the light wavelength in vacuum. The dots represent the signal intensity in the backward direction T_{14}. These theoretical results were obtained for $d_0=0.67$ μm, $d_1=0.68$ μm, and $d_2=0.26$ μm, $\varepsilon^{1/2}=2.8$ and Q of the order of 1000. Only the scale on the horizontal axis varies between the two parts (a) and (b).

$$Q'=\frac{\lambda_0}{2(\lambda''-\lambda_0)}, \tag{9}$$

where λ'' is the wavelength at which $T_{12}(\lambda'')=1/2$, such that $|\lambda''-\lambda_0|\gg|\lambda'-\lambda_0|$. We find, with a good approximation, that

$$Q'=\frac{\pi}{2}(1+2n_0+2n_1+n_2). \tag{10}$$

For multiplexing applications, one needs small values of Q, in order not to perturb the particles with wavelengths close to λ_0.

So, as described above, our system enables us to determine completely and in closed form all the wire lengths for a complete channel transfer, once wavelength λ_0 and the order of magnitude of the quality factor Q are chosen. Relations (6) give us the values of d_0, d_1, and d_2. Let us note that several solutions are possible due to the different values of the integers n_0, n_1, and n_2. Note that the $(-\delta)$ value in Eq. (6b) is associated with the $(+\delta)$ one in Eq. (6c) and vice

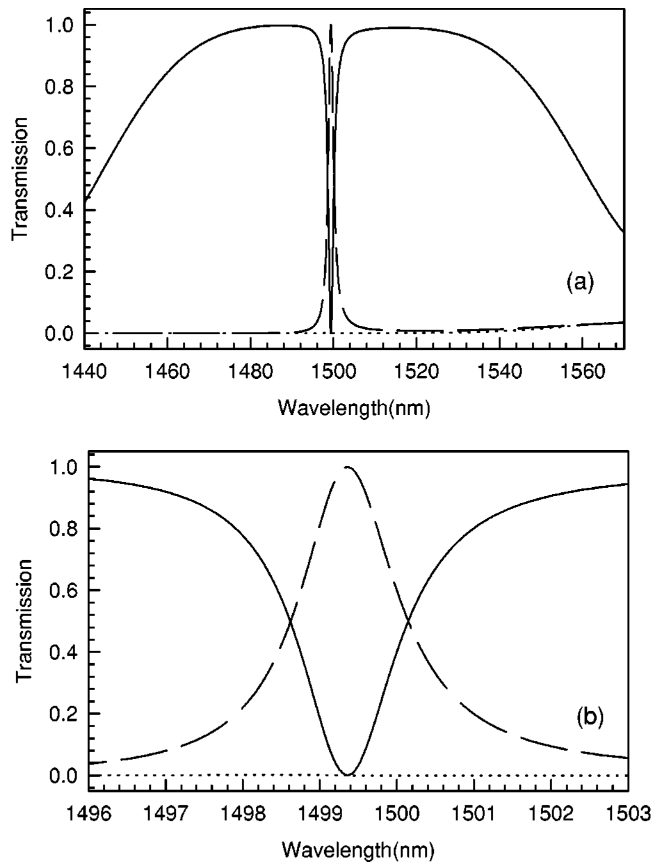

FIG. 3. Same as Fig. 2, but for $d_0 = 1.21$ μm, $d_1 = 1.22$ μm, and $d_2 = 0.52$ μm.

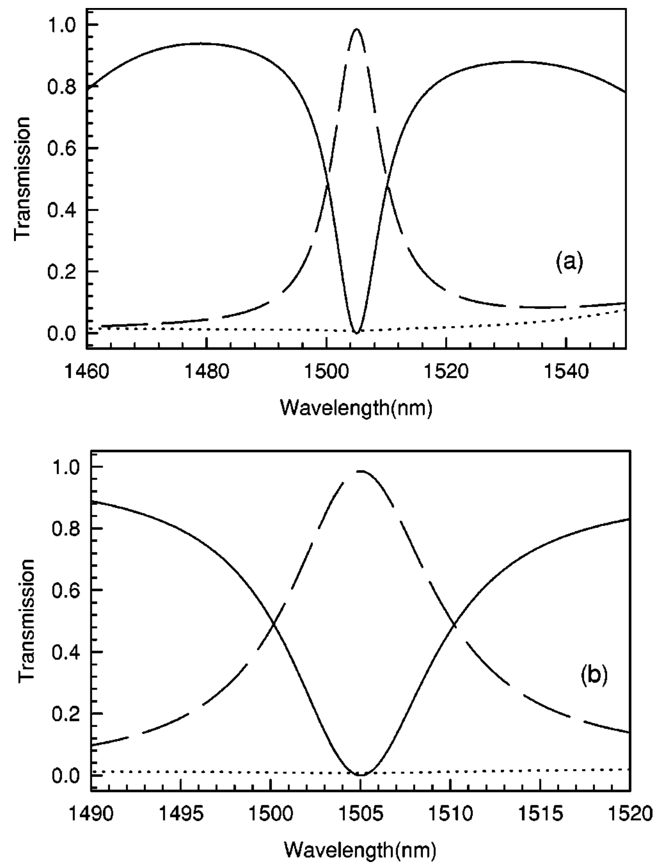

FIG. 4. Same as Fig. 3, but with Q of the order of 100, when $d_0 = 1.21$ μm, $d_1 = 1.25$ μm, and $d_2 = 0.49$ μm.

versa. Let us stress also that in order to have a multiplexer with a large nonperturbed region for the directly transmitted particle, one has to choose lengths d_0, d_1, and d_2 as small as possible. The smallest possible solution for a given λ_0 depends also on the width h of the monomode wires. It is possible to estimate that h has to be smaller than 0.5 μm for a vacuum wavelength $\lambda_0' = 1.5$ μm and a relative dielectric constant ε such that $\varepsilon^{1/2} = 2.8$ in order for the wires to be monomode. Current technologies enable one to prepare by electron lithography wires such that 0.2 μm$< h <$0.5 μm. With a width of 0.2 μm, one may choose $n_0 = n_1 = n_2 = 1$ in Eqs. (6) and to obtain a quality factor Q of the order of 1000 [Eq. (7)] we have $d_0 = 0.67$ μm, $d_1 = 0.68$ μm, and $d_2 = 0.26$ μm. We give these values with an accuracy of the order of 0.01 μm, which is currently available.

Let us note that once these lengths are fixed the wavelength and the quality factor of the transferred particle can be obtained from Eqs. (6b) and (6c) and (7) to be

$$\lambda_0 = \frac{4(d_1 + d_2)}{1 + 4n_1 + 2n_2} \tag{11}$$

and

$$Q = \frac{3}{\pi}(1 + 4n_1 + 2n_2)\frac{(d_1 + d_2)^2}{[2n_2 d_1 - (1 + 4n_1)d_2]^2}. \tag{12}$$

As can be seen in Fig. 2 obtained by an exact calculation of T_{12}, T_{13}, and T_{14} from Eqs. (1)–(4) as functions of the vacuum wavelength $\lambda' = \lambda(\varepsilon)^{1/2}$, the transferred particle wavelength $\lambda_0(\varepsilon)^{1/2}$, and quality factor Q are well estimated by Eq. (11) and (12), as well as the quality factor Q' [Eq. (10)] giving the unperturbed wavelength domain in T_{12}. Figure 3 shows mainly how an increase in the device size modifies the region of unperturbed transmission T_{12}. A comparison between Figs. 3 and 4 shows the effect of an increase in the width of the transferred frequency peak.

If one desires to shift λ_0, it is possible to adapt a heating device under the structure. Indeed on heating, all the distances will increase in the same proportion and λ_0 will increase proportionally to $(d_1 + d_2)$ as shown in Eq. (11). Another way to tune this multiplexer is to change the relative dielectric constant ε, as Eqs. (6) giving the distances are proportional to $\lambda_0'/(\varepsilon)^{1/2}$. This can be done by insolation techniques or by the application of an electric field.

Let us finally discuss the tuning accuracy required by the structure studied here. The numerators and denominators of the expressions given by Eqs. (3) can be developed to second order in $(\pi\delta/2)$ and to first order in $(\lambda - \lambda_0)$. One obtains

$$y_2 = 1 + 2\pi\frac{(\lambda_0 - \lambda)}{\lambda_0^2}(d_1 + d_2 + d_0) + \cdots, \tag{13a}$$

$$y_1 = y_2 - \frac{2(\pi\delta/2)^2 + \cdots}{(\pi\delta/2)^2 + 6\pi\lambda_0^{-2}(\lambda_0 - \lambda)(d_1 + d_2) + \cdots}, \tag{13b}$$

$$y_3 = -1 + 2\pi\frac{(\lambda_0 - \lambda)}{\lambda_0^2}(d_1 + d_2 + d_0) + \cdots, \tag{13c}$$

$$y_4 = y_3 + \frac{2(\pi\delta/2)^2 + \cdots}{(\pi\delta/2)^2 - 6\pi\lambda_0^{-2}(\lambda_0 - \lambda)(d_1 + d_2) + \cdots}. \tag{13d}$$

We see in the above expressions that for $\lambda = \lambda_0$, conditions (5) are satisfied for any small value of δ^2. In the structures studied before [3], this is not the case: five different lengths d_0, d_1, d_2, d_3, and d_4 are needed and similar expansions show that the fulfilment of conditions (5) requires much more precision for the different lengths, than for the present structure. More precisely for the results given by Fig. 2 of Ref. [3], in the numerators of Eqs. (13b) and (13d), one obtains terms of the order of δ^6 and in the denominators of these equations terms of the order of δ', where δ' is another small quantity associated with lengths d_3 and d_4, δ being associated with d_1 and d_2. As at the resonance, these δ^6 and δ' terms have to be equal; one understands that the precision on the lengths associated with this former structure has to be much higher than for the present one.

In the experimental realization of the structure studied here, the different lengths d_0, d_1, and d_2 can be made out of the same mask. So we expect that the relative imprecision between the two d_0, the eleven d_1 and the six d_2 of this structure can be neglected in front of the imprecision of the lengths d_0, d_1, and d_2 of the corresponding masks. So once we obtained the values of d_0, d_1, and d_2 from expressions (6), we varied them by a random amount and computed the transmission factors from the exact expressions (1)–(4). This procedure shows that the structure studied here functions with a precision of the order of 0.01 μm on the different lengths d_0, d_1, and d_2, for quality factors Q of a few thousands. Let us also remember that if the errors on d_1 and d_2 have the same sign, the shift in λ_0 may be of the order of 0.02 μm [see Eq. (11)]. But as explained above, λ_0 may be tuned back to its required value with the help of a heating device or by adjustment of the relative dielectric constant with an electric field.

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